Univ. of Calif.

Semi-Annual Status Report March 1, 1965 - August 31, 1965

"Constitutive Equations and non-Newtonian Fluid Mechanics"

N.A.S.A. Grant NsG-705/05/04-004

This report covers both the experimental and theoretical aspects of work in the slow flow of non-Newtonian fluids which have been carried out during the last six months.

1. <u>Theory:</u> A paper dealing with Stokes flow of Newtonian fluids entitled "Stokes Resistance - Translation and the Electrostatic Potential" was submitted to the Journal of Fluid Mechanics. As a result of a review by this journal the paper is being revised, and will be resubmitted in the near future.

A surprisingly compact form of the equation of motion for a Rivlin-Ericksen fluid of the second grade has been found.

$$\rho \bar{V}^{(2)} = -\nabla p^* + \mu_o \nabla^2 \bar{V} + (2\varphi_2 + \varphi_3)\left(\frac{\partial \nabla^2 \bar{V}}{\partial t} + \nabla^2 \bar{\omega} \times \bar{V}\right)$$

$$+ (\varphi_2 + \varphi_3)\left(\nabla^2 \bar{V}^{(2)} - 2 \frac{D}{Dt} \nabla^2 \bar{V}\right).$$

where

$$p^* = p - \frac{1}{2}(2\varphi_2 + \varphi_3)\left(\nabla^2 (V_2) - \omega^2\right)$$

and $\bar{V}^{(2)}$ is the acceleration. The stress tensor for this fluid is

$$\bar{\bar{\sigma}} = -\bar{\bar{I}}p + \mu_0 A^{(1)} + \varphi_2 A^{(2)} + \varphi_3 (A^{(1)})^2$$

The above equation of motion is most useful to perturbation theory as will be discussed below. It is interesting to note that in a steady flow the term whose coefficient is $2\varphi_2 + \varphi_3$ is a force which is always normal to the streamlines. If the Weissenberg symmetry rule for normal stresses in simple shear is valid then $2\varphi_2 + \varphi_3 = 0$, and the normal force vanishes.

A generalization of the Lorentz reciprocal theorem of Newtonian Stokes flow has been developed. With this theorem it can be shown that the $(n+1)^{th}$ approximation to the dissipation is determined entirely in terms of $\overline{V}_n$ the n^{th} approximation to the velocity field.

As an example of the power of this theorem we consider a particle translating with velocity $\overline{U}$ in an infinite non-Newtonian fluid. The velocity field is expanded in the form

$$\overline{V} = \overline{V}_0 + \lambda \overline{V}_1 + \lambda^2 \overline{V}_2 + \ldots$$

The stress tensor is approximated to the appropriate order by the theory of Rivlin-Ericksen fluids, and the drag force will be given by

$$\overline{F} = \overline{F}_0 + \overline{F}_1 + \overline{F}_2 + \ldots$$

In each case subscript $^{\circ}$ refers to the Newtonian problem. Using the reciprocal theorem and the above equation of motion it can be shown that

$$\overline{F}_1 \cdot \overline{U} = -3 \left(\frac{\varphi_2 + \varphi_3}{\mu_0} \right) \int_V p_0\, \omega_0^2 \, dv$$

Hence $\overline{F}_1$ is determined entirely by the pressure p_0 and vorticity $\overline{\omega}_0$ fields of the Newtonian problem. By inspection we can see that for symmetric particles such as spheres and ellipsoids $\overline{F}_1$ must vanish. However, for particles without fore and aft symmetry such as hemispheres, doublets of different size spheres, and cones $\overline{F}_1$ is non-zero. The importance of this result is that with appropriate corrections for finite boundaries it should be possible to determine $\varphi_2 + \varphi_3$ from drag force measurements with asymmetric particles. The integral for $\overline{F}_1$ is now being evaluated for doublets of two spheres and the hemisphere.

The equation of motion for a Rivlin-Ericksen fluid of the third grade is being investigated.

It is hoped to bring together these results in a paper during the next few months.

2. <u>Experiment:</u> A study of the fall times of spheres and other particles in a solution of polyisobutylene in white oil has been carried out by Mr. D. C. Wilson as an M.S. thesis. The solution was found to be remarkably homogeneous as determined by observing the fall of a small sphere through 10 cm. intervals for a total of 100 cm. (total fall time about 1500 secs.). However, the zero shear viscosity fluctuated about an average of $\pm$ 1% (a change that would be difficult to detect with conventional viscometry). These fluctuations appeared to depend on recent stress and temperature history, however other complicating factors were also observed. It was found to be necessary to pre-wet the particles since small air bubbles tended to cling to their surface if they were introduced dry. When lighting conditions were right refractive index patterns along the stream lines could be detected. A possible explanation of this effect can be found in the pre-wetting of the particles. Two fluid surfaces of the same

polymer solution when brought together do not instantaneously coalesce into a homogeneous phase. Even though the boundary will no longer be visible because of the continuity of the solvent a finite time is required for the discontinuity to disappear in the polymer. Such discontinuities could give rise to the refractive index patterns observed when pre-wetted particles are introduced into the solution. Clearly, this effect would give rise to variable mechanical properties. which might explain the scatter in the data. A new column in which particles and fluid are sealed in with no free surfaces is being constructed, and thus it should be possible to eliminate this effect. It has been concluded that in general free surfaces should be avoided in the study of polymer solutions since, despite precautions, evaporation takes place and "skins" are formed. These skins are almost insoluble and interfere drastically with experiments in the bulk of the fluid since they are carried into it by bodies introduced through the surface. Despite these difficulties some reproducible data have been obtained. These experiments are being continued in order to develop reliable techniques for obtaining highly precise reproducible fall times.